A Note About Localized Photons on the Brane

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Abstract

A first order formulation for the Maxwell field in five dimensions is dimentionally reduced using the Randall-Sundrum mechanism. We will see that massive photons can not be localized on the brane.

In the Randall-Sundrum mechanism[1], gravitons and scalar fields in $D = d + 1$ dimensions described by the Einstein-Hilbert and Klein-Gordon actions respectively, lead to gravitons and scalar fields localized on a d-dimensional brane. But, the usual (second order) Maxwell action can not yield photons trapped on the brane[2]. A solution of this dilemma was achieved in ref.[3] where the photons localized on the brane in four dimensions come from two antisymmetric tensors in five dimensions, satisfying a first order self-dual action[4]. Odd dimensional self-dual actions can be obtained from Kaluza-Klein dimentional reduction [5]. Specifically, Lu and Pope[3] found that the bosonic sector of ungauged $N = 2$, $D = 4$ supergravity can be obtained from $N = 4$, $D = 5$ gauged supergravity. The supersymmetric extension was considered by Duff, et al.[6] . The generalization for higher antisymmetric fields (p-forms) gives some negative results. Indeed, if we consider the Randall-Sundrum ansatz for the metric

$$ds^2 = e^{-2k|z|} g_{mn(x)} dx^m dx^n + dz^2, \tag{1}$$

where $m, n = 0, 1, ..., d$, together with the natural anzatz for a p-form

$$A_{M_1 ... M_p(x,z)} = A_{M_1 ... M_p(x)}, \tag{2}$$

then, the p-form is localized on the brane if

$$p < \frac{d-2}{2}. \tag{3}$$

For $d = 4$ (five dimensions), we have $p < 1$ and only scalar fields (0-form) can be trapped on the brane. But, a scalar field is dual to a 3-form in five dimensions, which is not localized on the brane if the anzatz (2) is considered. In consequence, apparently, the Randall-Sundrum mechanism can not explain the dual equivalence of p-forms. However, Duff and Liu [7] have solved this dilemma. They found that the right anzatz for p-forms is given by

$$A_{m_1 \ldots m_{p-1} z(x,z)} = e^{-2(p-\frac{d}{2})k|z|} A_{m_1 \ldots m_{p-1}(x)}, \quad A_{m_1 \ldots m_p (x,z)} = 0. \tag{4}$$

With this anzatz, the duality between p-form and $(D-p-2)$-form in the bulk implies the duality between $(p-1)$-form and $(D-(p-1)-2)$-form on the brane. The criterion for consistency in this case is

$$p > \frac{d}{2}. \tag{5}$$

If $d = 4$, we see that photons are excluded too. Thus, in five dimensions, p-forms with $p = 0$ and $p > 2$ can be trapped on the brane, ruled out the possibility of bounding photons (1-form) and its dual partner (2-form) on the brane[7]. The consistency with the Einstein equation will impose additional restrictions [7]. These results are based on the consideration of a second order action for p-forms. If we try to modify the ansatz (2) for vectors, including an explicit dependence with z in the following way: $A_M = (e^{-ak|z|}A_m, 0)$[6], then the action for massive vectors is obtained. In this note, we will see that not only massless vector fields but massive vector fields can not be localized on the brane in the framework of Randall-Sundrum dimentional reduction.

Let us start with the following first order action in five dimensions

$$S = -\frac{1}{12}\int d^4x dz [\epsilon^{MNPQR} H_{MNP} F_{QR} + \sqrt{-g} g^{MQ} g^{NR} g^{PS} H_{MNP} H_{QRS}], \tag{6}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$. H_{MNP} and A_M are independent fields. We can introduce the dual of H_{MNP} ($= \frac{1}{2}\sqrt{-g}\epsilon_{MNPQR} f^{QR}$) and (6) becomes the usual first order formulation for the Maxwell action. Their equations of motion are

$$H^{MNP} = -\frac{1}{2\sqrt{-g}} \epsilon^{MNPQR} F_{QR} \tag{7}$$

and

$$\epsilon^{MNPQR}\partial_N H_{PQR} = 0. \tag{8}$$

Substituting eq. (7) into action, we obtain the second order Maxwell action. On the other hand, eq. (8), can be solved (locally)

$$H_{MNP} = \partial_M B_{NP} + \partial_N B_{PM} + \partial_P B_{MN} \tag{9}$$

and substituting in the action, it becomes the action for an antisymmetric field (B_{MN}). In other words, the action (6) show us the dual equivalence between A_M and B_{MN} in five dimensions.

Now, we apply the Randall-Sundrum anzatz for the metric (eq. (1)). The ansatz for the H_{MNP} field is

$$H_{mnz(x,z)} = e^{-k|z|}b_{mn(x)}, \quad H_{mnp(x,z)} = 0 \tag{10}$$

and for the vector field

$$A_{m(x,z)} = e^{-k|z|}A_{m(x)}, \quad A_{z(x,z)} = 0. \tag{11}$$

The following reduced action is obtained

$$S = \int dz e^{-2k|z|}\int d^4x[-\frac{1}{4}\epsilon^{mnpq}b_{mn}F_{pq} - \frac{1}{4}\sqrt{-g_4}b_{mn}b^{mn}]. \tag{12}$$

The (auxiliary) field b_{mn} can be eliminated using its equation of motion

$$b^{mn} = -\frac{1}{2\sqrt{-g_4}}\epsilon^{mnpq}F_{pq} \tag{13}$$

and the Maxwell action trapped on the brane is obtained

$$S = \int dz e^{-2k|z|}\int d^4x[-\frac{1}{4}\sqrt{-g_4}F_{mn}F^{mn}]. \tag{14}$$

Although the Maxwell action appear on the brane, the ansatz is not consistent with the Einstein equation with cosmological constant ($\Lambda = -6k^2$). Indeed, we have the Einstein equation

$$R_{MN} - \frac{1}{2}g_{MN}R = g_{MN}\Lambda + T_{MN} \tag{15}$$

where

$$T_{MN} = -\frac{1}{2} g^{PR} g^{QS} H_{MPQ} H_{NRS} + \frac{1}{12} g_{MN} H_{PQR} H^{PQR}. \tag{16}$$

Substituting the ansatz (1) and (10), we find for the mn components

$$R_{4mn} - \frac{1}{2} g_{mn} R_4 = g^{pq} F_{mp(A)} F_{nq(A)} - \frac{1}{4} g_{mn} g^{pr} g^{qs} F_{pq(A)} F_{rs(A)} \tag{17}$$

where we have used eq. (13). Note that both sides of eq. (17) depend only of the coordinates x. Taking trace, we have $R_4 = 0$. Then, the zz component leads to an inconsistency

$$F_{mn} F^{mn} = 0. \tag{18}$$

Then, there is no consistency with the Einstein equation. This situation is similar to what happen in the usual Kaluza-Klein dimentional reduction if the zz component of the metric is $g_{zz} = 1$, instead of having $g_{zz} = \phi$. But the Randall-Sundrum ansatz does not admit any scalar field. Moreover, Duff and Liu [7] have shown that massless scalar and third rank antisymmetric fields, in five dimensions are the p-forms compatible with the Randall-Sundrum dimentional reduction when the coupling with gravity is considered. Furthermore, the ansatz $H_{mnp(x,z)} = 0$ is inconsistent with the equations of motion (7) and (8). Then, we must modify the anzatz for $H_{mnp(x,z)}$. This will lead to the action of a massive vector field on the brane, but the same inconsistency in the Einstein equations will be present. For instance, we choose

$$H_{mnp(x,z)} = e^{-2k|z|}(\partial_m B_{np(x)} + \partial_n B_{pm(x)} + \partial_p B_{mn(x)}) \equiv e^{-2k|z|} h_{mnp(x)}, \tag{19}$$

where $B_{mn(x)}$ is a second rank antisymmetric field. The following reduced action on the brane is obtained

$$S = \int d^4x dz e^{-2k|z|}[-\frac{1}{4}\sqrt{-g} F_{mn} F^{mn} - \frac{1}{12} h_{mnp} h^{mnp} - \frac{1}{4}\mu_z \epsilon^{mnpq} B_{mn} F_{pq}] \tag{20}$$

where $\mu_z = k e^{-k|z|}$ is a mass parameter. We identify the Cremmer-Sherk action [8] on the brane, which describes massive vector fields in a gauge invariant way. This action can be obtained from Kaluza-Klein dimentional reduction [9]

The mn components of the Einstein equation are now

$$
\begin{aligned}
R_{4mn} - \frac{1}{2} g_{mn} R_4 &= g^{pq} F_{mp(A)} F_{nq(A)} - \frac{1}{4} g_{mn} g^{pr} g^{qs} F_{pq(A)} F_{rs(A)} \\
&- \frac{1}{2} g^{pr} g^{qs} h_{mpq} h_{nrs} + \frac{1}{12} g_{mn} h_{pqr} h^{pqr}.
\end{aligned} \tag{21}
$$

Taking trace, the following value of the scalar curvature is obtained

$$
R_4 = \frac{1}{6} h_{mnp} h^{mnp} \tag{22}
$$

Using this value of R_4, the zz component of the Einstein equation yields the same inconsistency found previously, i.e., $F_{mn}F^{mn} = 0$. The same result is obtained if another ansatz for H_{mnp} is considered, e.g., $H_{mnp} = f_{(z)}\sqrt{-g}\epsilon_{mnpq}A^q$.

Summarizing, we have considered a first order formulation for the Maxwell action in five dimensions and we have seen that massive photons can not be localized on the brane in a consistent way, using the Randall-Sundrum mechanism.

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